

# MITO MEASUREMENTS OF THE SUNYAEV-ZELDOVICH EFFECT IN THE COMA CLUSTER OF GALAXIES

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## ABSTRACT

We have measured the Sunyaev-Zeldovich effect towards the Coma cluster (A1656) with the MITO experiment, a 2.6-m telescope equipped with a 4-channel 17 arcminute (FWHM) photometer. Measurements at frequency bands  $143\pm 15$ ,  $214\pm 15$ ,  $272\pm 16$  and  $353\pm 13$  GHz, were made during 120 drift scans of Coma. We describe the observations and data analysis that involved extraction of the SZ signal by employing a spatial and spectral de-correlation scheme to remove a dominant atmospheric component. The deduced values of the thermal SZ effect in the first three bands are  $\Delta T_0 = -179\pm 38$ ,  $-33\pm 81$ ,  $170\pm 35$   $\mu$ K in the cluster center. The corresponding optical depth,  $\tau = (4.1\pm 0.9)\times 10^{-3}$ , is consistent (within errors) with both the value from a previous low frequency SZ measurement, and the value predicted from the X-ray deduced gas parameters.

*Subject headings:* cosmology: cosmic microwave background – observations – galaxies: clusters: individual (A1656)

## 1. INTRODUCTION

Compton scattering of the cosmic microwave background (CMB) radiation by electrons in the hot gas in clusters of galaxies – the Sunyaev-Zeldovich (SZ) effect – has long been recognized as a uniquely important feature, rich in cosmological and astrophysical information (Zeldovich & Sunyaev 1969; Sunyaev & Zeldovich 1970; Boynton & Melchiorri 1978; Gunn 1978; Silk & White 1978; Cavaliere et al. 1979).

Interest in the effect heightened when high quality

images of the effect were obtained with interferometric arrays (Carlstrom et al. 2001; for general reviews, see Rephaeli 1995a, Birkinshaw 1999).

Accurate determinations of the Hubble constant ( $H_0$ ), cluster mass profiles, and cluster peculiar velocities from SZ and X-ray measurements require precise description of the intracluster (IC) gas temperature and density profiles, and improved control of systematic errors. These can be *more optimally* achieved by multi-frequency SZ measurements, and high-quality spectral and spatial X-ray measurements of *nearby* clusters.

Therefore, it is essential to develop sensitive SZ experiments operating at telescopes with effective beam sizes of a few arcminutes, equipped with bolometer arrays operating at frequencies on both sides of the Planckian peak. One such ground-based system is MITO (Millimetre and Infrared Testagrigia Observatory; De Petris et al. 1996).

Coma is the richest nearby cluster whose IC gas properties have been determined from extensive X-ray observations. Attempts to measure the S-Z effect in Coma at low frequencies were reported by Pariskii (1973), Rudnick (1978), Lake & Partridge (1980), Birkinshaw et al. (1981), and Herbig et al. (1995, revised in Mason et al. (2001))

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TABLE 1  
(SEE TEXT FOR EXPLANATION)

Channel $i$	$\nu_i$ (GHz)	$\langle a \rangle_i$ 2000	$\langle a \rangle_i$ 2001	$b_i$	$c_i$	$A\Omega_i$ (cm <sup>2</sup> sr)	$R_i$ ( $\mu$ K/nV)
1	$143\pm 15$	1	1	1	1	$0.40\pm 0.02$	$433\pm 40$
2	$214\pm 15$	$1.51\pm 0.45$	$2.59\pm 1.54$	$0.11^{+0.01}_{-0.01}$	$1.18^{+0.00}_{-0.07}$	$0.36\pm 0.02$	$371\pm 35$
3	$272\pm 16$	$2.96\pm 1.21$	$4.96\pm 2.75$	$-0.93^{+0.11}_{-0.00}$	$1.08^{+0.01}_{-0.08}$	$0.34\pm 0.02$	$400\pm 40$
4	$353\pm 13$	$12.42\pm 2.60$	$15.98\pm 8.76$	$-1.45^{+0.44}_{-0.24}$	$0.89^{+0.12}_{-0.16}$	$0.34\pm 0.02$	$347\pm 40$

while at high frequencies by Silverberg et al. (1997).

## 2. OBSERVATIONS

### 2.1. MITO

MITO is a 2.6-m alt-azimuthal telescope located at an altitude of 3480 m in the Italian Alps. The telescope design, optical configuration, and electro-mechanical modulation system are described in De Petris et al. 1989, Gervasi et al. 1998, Mainella et al. 1996. The 4-channel photometer (Orlando et al. 2002) consists of 4 NTD Ge composite bolometers cooled down to 290 mK. The thermodynamic NET is around 1 mK s<sup>1/2</sup>. At the telescope focal plane the photometer has a 17' FWHM field of view (FOV).

Observations were made during February and March of 2000 & 2001, when data from some 120 drift scans of Coma were collected. Each scan consists of 10 minutes of measurements collected every second. A 41' beamthrow was selected for a 3-field square-wave like spatial modulation. The position of Coma in the sky covered altitude values from 50 up to 73 degrees above the horizon. The drift scan systematics are better controlled than in source tracking mode, mainly stemming from weaker microphonics and lower changes in sidelobe spillover effects. At a data sampling rate of 1 Hz, pointing information, signals, modulation system and photometer performance were recorded. A pointing accuracy of  $\sim 1'$  was attained by frequent measurements of bright stars close to the source by a CCD camera. We measured beam shape, responsivity and atmospheric transmission for each channel by means of planetary calibrators – Jupiter, Saturn and the Moon – before each Coma observation session. In order to correct for atmospheric absorption we measured the channels zenith transmission during each night.

### 2.2. Results from X-ray Observations

Coma has been extensively observed, mostly in the 1-10 keV range. The most recent high spectral and spatial resolution measurements with the XMM/EPIC experiment are best fit by  $kT=8.2\pm0.1$  keV (at 90% confidence level) in the central 10' region (Arnaud et al. 2001). Analysis of the emission from a larger region of the cluster yields  $kT=8.2\pm0.4$  keV, which we adopt here. This value of the temperature is in very good agreement with the value deduced from Ginga observations. Since quantitative results for the gas density profile from neither XMM nor *Chandra* are currently available, we use the ROSAT (Briel, Henry, & Bohringer 1992) deduced values  $n_{e0} \simeq (2.89 \pm 0.04) \times 10^{-3}$  cm<sup>-3</sup>,  $r_c \simeq 10.5' \pm 0.6'$ , and  $\beta \simeq 0.75$ , for the central electron density, core radius, and index in the expression for the commonly used  $\beta$  density profile,  $n_e(r) = n_{e0}(1 + r^2/r_c^2)^{-3\beta/2}$ .

## 3. DATA ANALYSIS

Clearly, there is no unique approach to the analysis of a noisy dataset which includes a dominant, fluctuating atmospheric emission, and other confusing signals. When the spatial distribution of the confusing source is unknown, such as that of CMB anisotropy, a sufficient number of channels is selected so as to allow removal of anticipated foregrounds but the corresponding system of equations can often be numerically unstable.

Our task is made feasible largely by the known position of the source, and its approximate angular extent. Therefore, to first approximation, we can fit the data with a signal of a known shape but unknown peak amplitude. The errors in measuring this amplitude approach the intrinsic detector noise if we can successfully remove the contributions of atmospheric fluctuations and CMB anisotropy whose spectral shape is known. In addition to the SZ effect, these signals were modeled separately from all others which were collectively treated as unidentified noise. While three channels are in principle sufficient to achieve this goal, we added a fourth channel centered on the crossover frequency (where the thermal SZ effect vanishes), in order to check the separation between CMB and atmospheric emission, and to *possibly* obtain a rough estimate on the value of the kinematic SZ effect.

We have computed the expected spectral ratios of the SZ signals in channels 2, 3, and 4 with respect to the first channel. The calculation is relativistically exact and is based on the treatment of Rephaeli (1995b; see also Rephaeli & Yankovitch 1997), taking the above quoted range of the gas temperature. At each frequency we calculated the intensity change due to the thermal SZ effect,  $\Delta I_{sz}$ , and convolved it with the spectral response of the photometer,  $\epsilon_i$ , and the atmospheric transmittance  $\epsilon_{atm}$ , which was determined from a model (Liebe 1985). The expected ratio of the  $i$ -th to the first channel is thus,

$$b_i = \frac{A\Omega_i R_i \int_0^\infty \Delta I_{sz}(\nu) \epsilon_i(\nu) \epsilon_{atm}(\nu) d\nu}{A\Omega_1 R_1 \int_0^\infty \Delta I_{sz}(\nu) \epsilon_1(\nu) \epsilon_{atm}(\nu) d\nu}, \quad (1)$$

where  $R_i$  is the responsivity in the  $i$ -th channel as measured with sky calibrators and  $A\Omega_i$  is the throughput, which slightly changed from channel to channel due to the different optical paths through the telescope and photometer. Uncertainties in the evaluation of the  $b_i$  are due to the relatively small error in the value of  $kT$  and the fluctuating values of atmospheric transmittance (0.5-2.0 mm of precipitable water vapour). Similarly, we have calculated the ratios,  $c_i$ , of the intensity change due to the primary CMB anisotropy in the four bands. In Table 1 all the cited ratios are listed with  $R_i$  and  $A\Omega_i$ .

The recorded data in the four channels ( $i=1,...,4$ ) for each scan ( $j=1,...,120$ ) can be written as follows:

$$\begin{aligned} \Delta S_i^j &= a_i^j \Delta V_{atm}^j(t) + b_i \Delta V_{sz}^j(\alpha, \delta) \\ &+ c_i \Delta V_{cmb}^j(\alpha, \delta) + \Delta V_{offset,i}^j(\alpha, \delta; t) \\ &+ \delta V_{spike,i}^j(t) + \Delta V_{noise,i}^j(t). \end{aligned} \quad (2)$$

In this equation,  $b_i \Delta V_{sz}^j$  is the SZ signal which is fit with  $w_i^j \Delta V_{sz-sim}^j$ , where  $\Delta V_{sz-sim}^j$  is the simulated SZ signal, normalized to 1 at the peak;  $w_i^j$  are the estimated peak values of SZ signals (in the absence of noise  $\langle w_1 \rangle = b_1^{-1} \langle w_i \rangle$ ,  $i = 2, 3, 4$ ) and  $a_i^j$  are the atmospheric ratios with respect to channel 1, to be determined for each scan. The  $a_i^j$  values could change from scan to scan due to the atmospheric long time variations while they are assumed to be constant within a single scan. Since atmospheric fluctuations dominate on each scan, a rough estimate of the  $a_i^j$  can be done as the ratios of the channels standard deviations. For this purpose we used only the

data outside the expected position of Coma. In Table 1 we have also quoted the mean values of  $a_i$  averaged over all the drift scans in each observational campaign. Detector and other sources of noise are collectively included in the term  $\Delta V_{noise,i}^j$ . Estimated errors in the evaluation of  $w_i^j$  due to uncertainties in  $b_i$  constitute a small fraction of the overall error.

The data include three main sources of contamination:

**a.** Cosmic ray spikes ( $\delta V_{spike,i}^j(t)$ ): we observed roughly one spike in every 10 scans. A special de-glitching algorithm has been applied to correct these data. However, their total removal from the analysis did not change our results apart from a slight increase of the noise. After removal of the spikes the data were averaged over 15 seconds so that each drift scan consists of 37 bins roughly separated in sky by 3.75 arcminutes, about 1/4 of the FOV.

**b.** An instrumental offset ( $\Delta V_{offset,i}^j(\alpha, \delta; t)$ ) ranging from 0.4  $\mu$ V (channel 1) up to 5  $\mu$ V (channel 4) was measured. This is partly due to the residual side lobes of the telescope, and partly to the thermal imbalance of the primary mirror. The telescope is stopped during each drift scan. In this way the offset is obviously independent of position, so it is possible to remove it from each scan through a linear fit of the data; a quadratic fit did not change the rms value of the residual fluctuations. Note that the removal of the offset through a linear fit may introduce an additional effect, due to the presence of large atmospheric fluctuations. The average of these along a finite drift scan will introduce an additional offset,  $\Delta V_{off-atm,i}^j = \frac{1}{37} \sum_{k=1}^{37} a_i^j \Delta V_{atm,k}^j(t)$ . This unknown offset will in turn introduce an error in the estimated value of the SZ signal; we discuss below how this offset was removed.

**c.** Large atmospheric fluctuations; we based our approach to minimize their effect on the measured data in channel 4, based on the fact that  $a_4$  is  $\sim 13$ . The efficiency of this procedure depends on the amplitude of the atmospheric fluctuations in each drift scan, as well as on the degree of correlation among the various channels with channel 4. A detailed discussion of the atmospheric noise will be given elsewhere; here we briefly discuss only the essential points. In channel 2 the residuals of the signal after subtracting  $a_2^j/a_4^j \Delta S_4^j$  and minimizing the difference consist of uncorrected atmospheric fluctuations, signal due to CMB anisotropy, and only a very small SZ signal. If we fit the residuals with a simulated SZ signal of amplitude  $w_2^j$ , we get  $\langle w_2 \rangle \simeq -0.25 \pm 0.32$  nV. This result indicates that the SZ signal in channel 2 is consistent with zero, as expected, and the large dispersion – with respect to the final error due to the intrinsic detector noise of about 0.05 nV – around the mean suggests that there is a residual atmospheric contamination  $\Delta w_2 = a_2 \Delta w_1$ . A significant decrease in the dispersion is obtained if we remove the values of  $w_2$  far from the mean. These values are possibly due to the presence of a large atmospheric disturbance along the line of sight (los) to Coma. For instance, if we remove values which are more than  $4\sigma$  (standard deviation) from the mean, we obtain  $\langle w_2^* \rangle \simeq -0.04 \pm 0.09$  nV. A similar analysis has been done for channels 1 and 3, for which we expect SZ signals at the predicted ratio  $b_3$ . Residual atmospheric fluctuations – as well as the spurious offset discussed previously – introduce an additional error. The

corresponding small corrections  $\Delta w_1$  and  $\Delta w_3 = a_3 \Delta w_1$ , which have to be added to the measured values of the SZ effect in order to get the correct value, must satisfy the relation:  $w_1 + \Delta w_1 = (w_3 + a_3 \Delta w_1)/b_3$ . The dispersion of  $(w_1 + \Delta w_1)$  is minimized by changing  $b_3^*$  within  $\pm 1.5 b_3^*$ . The dispersion decreases by a factor of four when  $b_3^*$  coincides with  $b_3$  within the errors.

After correcting for binning and atmospheric transmittance we obtain  $\langle w_1^* \rangle \simeq -0.171 \pm 0.036$ ,  $\langle w_2^* \rangle \simeq -0.037 \pm 0.090$ , and  $\langle w_3^* \rangle \simeq 0.174 \pm 0.036$  nV. In Figure 1, we show plots of  $\Delta V_{sz}$ ,  $\Delta V_{cmb}$ , and  $\Delta V_{atm}$  as derived by solving eq.(2). The data have been filtered with an adaptive filter which removes spatial patterns that are not present in the expected SZ profile. The filter does not alter the SZ signal, but strongly reduces CMB and atmospheric fluctuations.

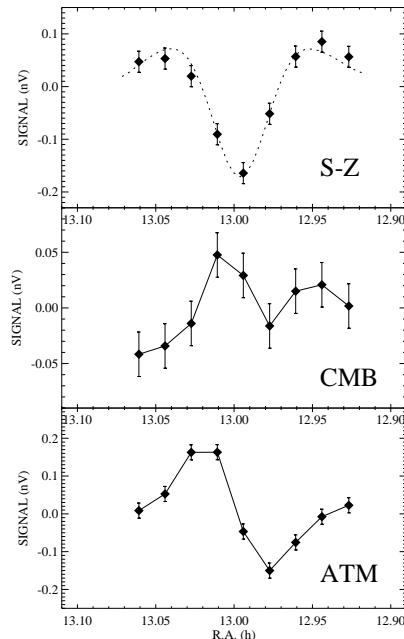


FIG. 1. SZ, CMB, and atmospheric signals extracted from equation (2), corresponding to the 143 GHz band, averaged over the drift scans and binned in 9 independent sky positions. Atmospheric fluctuations have an amplitude comparable to that of the SZ effect, even after 20 hours of integration. CMB anisotropies are averaged over various parallactic angles corresponding to the different drift scans.

The  $w$  value has been converted to thermodynamic temperature for the  $i$ -th channel by calibration based on measurements of Jupiter (Encarnaz & Moreno 2002). Integrating its spectral temperature distribution over the four channels we determined the optical responsivities. The overall calibration error is about 10%, dominated by the uncertainty in the brightness temperature of Jupiter ( $\sim 5\%$ ), and statistical errors in the estimate of the beamsize. The final values of the deduced temperature changes are  $\Delta T \simeq -73.5 \pm 15.5, -13.7 \pm 33.3, 69.6 \pm 14.4$   $\mu$ K, in the first three channels, respectively. Based on our calculated value for the geometrical form factor due to beam dilution and modulation strategy,  $\eta = 0.41 \pm 0.02$ , we compute the corresponding values  $\Delta T_0 \simeq -179.3 \pm 37.8, -33.4 \pm 81.2, 169.8 \pm 35.1$   $\mu$ K for  $\Delta T$  in the cluster center.

## 4. DISCUSSION

MITO measurements of the Coma are the first successful detection of the SZ effect in this cluster at high frequencies. Our results imply a mean Thomson optical depth  $\tau_0 = (4.1 \pm 0.9) \times 10^{-3}$  for a los through the center of Coma. Herbig et al. (1995) have already detected the effect at the much lower frequency of 32 GHz. Their reported central value,  $\Delta T_0 = -505 \pm 92 \mu\text{K}$ , and corresponding optical depth,  $\tau_0 = (5.6 \pm 1.1) \times 10^{-3}$ , were recently updated (Mason *et al.*, 2001) with new calibration data, yielding an observed decrement  $\Delta T = -302 \pm 54 \mu\text{K}$ . This value, corrected for the dilution over their  $7.3'$  FOV and the switching amplitude of  $22.2'$ , corresponds to  $\Delta T_0 = -520 \pm 93 \mu\text{K}$  and  $\tau_0 = (4.0 \pm 0.6) \times 10^{-3}$ , and is clearly in good agreement with X-ray deduced gas parameters. Clearly, these two results for  $\tau_0$  are quite consistent, given the appreciable ( $1\sigma$ ) errors. Even though our somewhat lower value is not significantly different from that of Herbig et al., it is interesting to note that the CMB anisotropy signal that we seem to have identified along a los to Coma (heavily exploiting our multi-frequency capability) could have possibly contributed to the SZ signal at the low (single) frequency OVRO measurement (where the two signals cannot be spectrally separated). Irrespective of whether this is indeed the case, the great advantage of multi-frequency work has been clearly demonstrated in the analysis of our measurements.

In Figure 2 we show the first SZ spectrum of Coma by combining OVRO with our measurements; a fit to the predicted spectrum yields  $\tau_0 \simeq (4.9 \pm 0.7) \times 10^{-3}$ .

In addition to measurement errors, other errors include the use of a simple one dimensional, isothermal gas density with a  $\beta$  profile. Based on the X-ray morphology of Coma, the relatively small degree of ellipticity of X-ray morphologies of other rich clusters, and the recent verification of the gas isothermality in Coma from XMM measurements (Arnaud et al. 2001), we estimate that the additional error introduced by taking the gas to be spherically symmetric and isothermal is  $\leq 30\%$  of the overall quoted error. Note also that the additional Comptonization by non-thermal electrons, (whose existence in Coma is deduced from measurements of extended radio emission) is negligible (Shimon & Rephaeli 2002, Colafrancesco, Marchegiani & Palladino 2001) if the energy density in these electrons is at the level deduced from measurements of high energy ( $> 20$

keV) X-ray emission with the RXTE (Rephaeli, Gruber & Blanco 1999) and BeppoSAX (Fusco-Femiano et al. 1999) satellites. This result is valid also for other *viable* energetic electron models (Shimon & Rephaeli 2002).

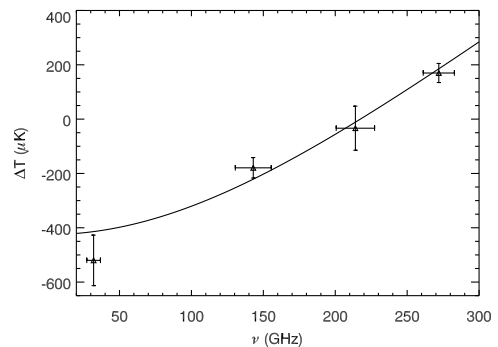


FIG. 2. SZ spectrum of the Coma cluster. The solid line shows the best fit spectrum (taking isothermal gas with  $kT = 8.2$  keV) to the combined MITO and OVRO (Mason *et al.* 2001) measurements, corresponding to  $\tau \simeq (4.9 \pm 0.7) \times 10^{-3}$ .

The work reported here is only the first stage of an extensive project to upgrade MITO to a bolometer array consisting of 36 elements operating at four frequency bands and with beamsizes down to  $4'$ . Work on the upgraded system is underway, with first observations scheduled for early 2003. We plan to measure the effect in a large sample of nearby clusters, with the goal of determining  $H_0$  and cluster masses. Based on the results from our long campaign to measure the effect in Coma, we are reasonably confident that this project is quite feasible.

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